

Mark Scheme 4723
January 2006

1	Obtain integral of form $k \ln x$	M1	[any non-zero constant k ; or equiv such as $k \ln 3x$]
	Obtain $3 \ln 8 - 3 \ln 2$	A1	[or exact equiv]
	Attempt use of at least one relevant log property	M1	[would be earned by initial $\ln x^3$]
	Obtain $3 \ln 4$ or $\ln 8^3 - \ln 2^3$ and hence $\ln 64$	A1 4	[AG ; with no errors]

2	Attempt use of identity linking $\sec^2 \theta$, $\tan^2 \theta$ and 1	M1	[to write eqn in terms of $\tan \theta$]
	Obtain $\tan^2 \theta - 4 \tan \theta + 3 = 0$	A1	[or correct unsimplified eqn]
	Attempt solution of quadratic eqn to find two values of $\tan \theta$	M1	[any 3 term quadratic eqn in $\tan \theta$]
	Obtain at least two correct answers	A1	[after correct solution of eqn]
	Obtain all four of 45, 225, 71.6, 251.6	A1 5	[allow greater accuracy or angles to nearest degree – and no other answers between 0 and 360]

3 (a)	Attempt use of product rule	M1	[involving ... + ...]
	Obtain $2x(x+1)^6 \dots$	A1	
	Obtain $\dots + 6x^2(x+1)^5$	A1 3	[or equivs; ignore subsequent attempt at simplification]
(b)	Attempt use of quotient rule	M1	[or, with adjustment, product rule; allow u/v confusion]
	Obtain $\frac{(x^2 - 3)2x - (x^2 + 3)2x}{(x^2 - 3)^2}$	A1	[or equiv]
	Obtain -3	A1 3	[from correct derivative only]

4 (i)	State $y \leq 2$	B1 1	[or equiv; allow <; allow any letter or none]
(ii)	Show correct process for composition of functions	M1	[numerical or algebraic]
	Obtain 0 and hence 2	A1 2	[and no other value]
(iii)	State a range of values with 2 as one end-point	M1	[continuous set, not just integers]
	State $0 < k \leq 2$	A1 2	[with correct < and \leq now]

5	Obtain integral of form $k(1-2x)^6$	M1	[any non-zero constant k]
	Obtain correct $-\frac{1}{12}(1-2x)^6$	A1	[or unsimplified eqn; allow + c]
	Use limits to obtain $\frac{1}{12}$	A1	[or exact (unsimplified) eqn]
	Obtain integral of form ke^{2x-1}	M1	[or equiv; any non-zero constant k]
	Obtain correct $\frac{1}{2}e^{2x-1} - x$	A1	[or equiv; allow + c]
	Use limits to obtain $-\frac{1}{2}e^{-1}$	A1	[or exact (unsimplified) eqn]
	Show correct process for finding required area	M1	[at any stage of solution; if process involves two definite integrals, second must be negative]
	Obtain $\frac{1}{12} + \frac{1}{2}e^{-1}$	A1 8	[or exact eqn; no + c]

<p>6 (a) <u>Either</u>: State proportion $\frac{440}{275}$ Attempt calculation involving proportion Obtain 704</p> <p><u>Or</u>: Use formula of form $275e^{kt}$ or $275a^t$ Obtain $k = 0.047$ or $a = \sqrt[10]{1.6}$ Obtain 704</p>	<p>B1</p> <p>M1 [involving multn and X value] A1 3</p> <p>M1 [or equiv] A1 [or equiv] A1 (3) [allow ± 0.5]</p>
<p>(b)(i) Attempt correct process involving logarithm</p> <p>Obtain $\ln \frac{20}{80} = -0.02t$ Obtain 69</p>	<p>M1 [or equiv including systematic trial and improvement attempt] A1 [or equiv] A1 3 [or greater accuracy; scheme for T&I: M1A2]</p>
<p>(ii) Differentiate to obtain $k e^{-0.02t}$ Obtain $-1.6e^{-0.02t}$ (or $1.6e^{-0.02t}$) Obtain 0.88</p>	<p>M1 [any constant k different from 80] A1 [or unsimplified equiv] A1 3 [or greater accuracy; allow -0.88]</p>
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<p>7 (i) Sketch curve showing (at least) translation in x direction Show correct sketch with one of 2 and 3π indicated ... and with other one of 2 and 3π indicated</p>	<p>M1 [either positive or negative] A1 A1 3</p>
<p>(ii) Draw straight line through O with positive gradient</p>	<p>B1 1 [label and explanation not required]</p>
<p>(iii) Attempt calculations using 1.8 and 1.9 Obtain correct values and indicate change of sign</p>	<p>M1 [allow here if degrees used] A1 2 [or equiv; $x = 1.8$: LHS = 1.93, diff = 0.13; $x = 1.9$: LHS = 1.35, diff = -0.55; radians needed now]</p>
<p>(iv) Obtain correct first iterate 1.79 or 1.78 Attempt correct process to produce at least 3 iterates Obtain 1.82</p>	<p>B1 [or greater accuracy] M1 A1 [answer required to exactly 2 d.p.; $2 \rightarrow 1.7859 \rightarrow 1.8280 \rightarrow 1.8200$; SR: answer 1.82 only - B2]</p>
<p>Attempt rearrangement of $3 \cos^{-1}(x-1) = x$ or of $x = 1 + \cos(\frac{1}{3}x)$ Obtain required formula or equation respectively</p>	<p>M1 [involving at least two steps] A1 5</p>

- 8 (i) Differentiate to obtain $kx(5 - x^2)^{-1}$ **M1** [any non-zero constant]
 Obtain correct $-2x(5 - x^2)^{-1}$ **A1** [or equiv]
 Obtain -4 for value of derivative **A1**
 Attempt equation of straight line through $(2, 0)$ with numerical value of gradient obtained from attempt at derivative **M1** [not for attempt at eqn of normal]
 Obtain $y = -4x + 8$ **A1 5** [or equiv]
- (ii) State or imply $h = \frac{1}{2}$ **B1**
 Attempt calculation involving attempts at y values **M1** [addition with each of coefficients 1, 2, 4 occurring at least once]
 Obtain $k(\ln 5 + 4\ln 4.75 + 2\ln 4 + 4\ln 2.75 + \ln 1)$ **A1** [or equiv perhaps with decimals; any constant k]
 Obtain 2.44 **A1 4** [allow ± 0.01]
- (iii) Attempt difference of two areas **M1** [allow if area of their triangle $<$ area A]
 Obtain $8 - 2.44$ and hence 5.56 **A1√ 2** [following their tangent and area of A providing answer positive]
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- 9 (i) State $\sin 2\theta \cos \theta + \cos 2\theta \sin \theta$ **B1**
 Use at least one of $\sin 2\theta = 2 \sin \theta \cos \theta$ and $\cos 2\theta = 1 - 2 \sin^2 \theta$ **B1**
 Attempt complete process to express in terms of $\sin \theta$ **M1** [using correct identities]
 Obtain $3 \sin \theta - 4 \sin^3 \theta$ **A1 4** [AG; all correctly obtained]
- (ii) State 3 **B1**
 Obtain expression involving $\sin 10\alpha$ **M1** [allow θ/α confusion]
 Obtain 9 **A1 3** [and no other value]
- (iii) Recognise $\operatorname{cosec} 2\beta$ as $\frac{1}{\sin 2\beta}$ **B1** [allow θ/β confusion]
 Attempt to express equation in terms of $\sin 2\beta$ only **M1** [or equiv involving $\cos 2\beta$]
 Attempt to find non-zero value of $\sin 2\beta$ **M1** [or of $\cos 2\beta$]
 Obtain at least $\sin 2\beta = \sqrt{\frac{5}{12}}$ **A1** [or equiv, exact or approx]
 Attempt correct process to find two values of β **M1** [provided equation is $\sin 2\beta = k$; or equiv with $\cos 2\beta$]
 Obtain 20.1, 69.9 **A1 6** [and no others between 0 and 90]